PERTH MODERN SCHOOL

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INDEPENDENT PUBLIC SCHOOL

Semester One Examination, 2020

Question/Answer booklet

MATHEMATICS SPECIALIST
UNIT 1

Section One:
Calculator-free

Your Name: SOLUTIONS

Your Teacher's Name:

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Marks	Max	Question	Mark	Max
1		4	5		8
2		6	6		12
3		5	7		6
4		5	8		4

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	46	35
Section Two: Calculator-assumed	12	12	100	100	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 11 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section One: Calculator-free **(46 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1 {1.1.1, 1.1.3} **(4 marks)**

(a) Solve $\frac{(n+1)!}{n(n-2)!} = 8$ (2 marks)

$$\frac{(n+1)(n)(n-1)(n-2)!}{n(n-2)!} = 8$$

$(n+1)(n-1) = 8$ ✓ simplifies

$n^2 - 1 = 8$

$n^2 = 9$

$n = 3$ ✓ positive value only

(b) Evaluate ${}^{15}P_9 \div {}^{12}P_8$ (2 marks)

$$\frac{15!}{6!} \div \frac{12!}{4!} = \frac{15!}{6!} \times \frac{4!}{12!}$$

$= \frac{\overset{1\,3}{\cancel{15}} \times \overset{7}{\cancel{14}} \times 13}{\underset{1\,2}{\cancel{6}} \times \underset{1}{\cancel{5}}}$ ✓ simplifies

$= 7 \times 13$

$= 91$ ✓ correct answer

Question 2 {1.3.1, 1.3.2, 2.3.1} **(6 marks)**

Consider the following statement:

If the diagonals of a quadrilateral are equal in length, then the quadrilateral is a square

(a) Write the converse of the statement and state whether it is true or false. (2 marks)

> If a given quadrilateral is a square, then its diagonals are equal in length. ✓
>
> True. ✓

(b) Write the inverse of the statement and state whether it is true or false. (2 marks)

> If the diagonals of a quadrilateral are not equal in length, then the quadrilateral is not a square. ✓
>
> True. ✓

(c) Write the contrapositive of the statement and state whether it is true or false. (2 marks)

> If a given quadrilateral is not a square, then its diagonals are not equal in length. ✓
>
> False. ✓

Question 3 {2.3.1} **(5 marks)**

a) Prove the following statement: (4 marks)

If n is 3 more than a multiple of 6, then n^2 is also 3 more than a multiple of 6.

Assume that n is 3 more than a multiple of 6.

Then $n = 6k+3$ for some $k \in \mathbb{Z}$. ✓ assumes condition and writes n as $6k+3$

Hence $n^2 = (6k+3)^2$

$= 36k^2 + 36k + 9$ ✓ expands expression

$= 6(6k^2 + 6k + 1) + 3$ ✓ writes in form $6m+3$

Since $6k^2+6k+1$ is an integer, n^2 is 3 more than a multiple of 6. ✓ concludes consequence

QED

b) Is there an integer m such that $\sqrt{6m+1}$ is equal to 3 more than a multiple of 6? Explain briefly. (1 mark)

No. If there were, then $(\sqrt{6m+1})^2 = 6m+1$ would also be 3 more than a multiple of 6, which it clearly is not.

✓ states 'No' with correct reason

Question 4 {2.3.3} **(5 marks)**

Prove by contradiction that $\sqrt{10}$ is irrational.

Assume that $\sqrt{10}$ is rational. ✓ Initial assumption

Then $\sqrt{10} = \frac{a}{b}$ where a and b are coprime integers. ✓ Implies that a & b are coprime

Rearranging,

$$a = \sqrt{10}b$$
$$a^2 = 10b^2$$
$$a^2 = 2(5b^2)$$

a^2 *is even.* ✓ states a is even when a^2 is even (proof not required)
a *is even.*

Hence, $a = 2n$ for some integer n.

Substituting,

$$10b^2 = (2n)^2$$
$$10b^2 = 4n^2$$
$$5b^2 = 2n^2$$

$5b^2$ *is even.* ✓ states b^2 is even when $5b^2$ is even (proof not required)
b^2 *is even, since 5 is odd.*
b *is even.*

Since a and b are both even, they cannot be coprime.

Therefore, $\sqrt{10}$ cannot be rational. ✓Explains why there is a contradiction

Question 5 {1.2.1-1.2.4, 1.2.6-1.2.9} **(8 marks)**

Points P and Q have position vectors of $-5\boldsymbol{i} + 2\boldsymbol{j}$ and $-2\boldsymbol{i} - \boldsymbol{j}$ respectively, relative to the origin O.

(a) Determine the unit vector of the displacement of Q relative to P in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$.

(3 marks)

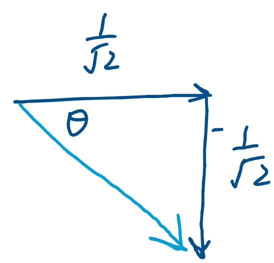
$$_{Q}\underset{\sim}{r}_{P} = \underset{\sim}{r}_{Q} - \underset{\sim}{r}_{P}$$
$$= -2\underset{\sim}{i} - \underset{\sim}{j} - (-5\underset{\sim}{i} + 2\underset{\sim}{j})$$
$$= 3\underset{\sim}{i} - 3\underset{\sim}{j}$$ ✓ relative displacement

$$|_{Q}\underset{\sim}{r}_{P}| = \sqrt{3^2 + (-3)^2}$$
$$= \sqrt{18} = 3\sqrt{2}$$ ✓ correct magnitude

$$_{Q}\hat{\underset{\sim}{r}}_{P} = \frac{1}{3\sqrt{2}}(3\underset{\sim}{i} - 3\underset{\sim}{j})$$
$$= \frac{1}{\sqrt{2}}(\underset{\sim}{i} - \underset{\sim}{j})$$ ✓ unit vector

(b) Give the direction of the vector in part (a) as a bearing if the unit vector $\boldsymbol{j}$ is pointing due North. (2 marks)

$$\tan\theta = -1$$



$$\theta = -45^\circ$$ ✓ finds angle

$$\text{Bearing} = 180^\circ + 45^\circ$$
$$= 135^\circ N$$ ✓ correct bearing

(c) Vector $\overrightarrow{OR}$ has magnitude $3\sqrt{2}$ and is parallel to the vector in part (a) and in the opposite direction. Determine $\overrightarrow{OR}$ in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$. (3 marks)

$$\overrightarrow{OR} = 3\sqrt{2}\left(-\frac{1}{\sqrt{2}}(\underset{\sim}{i} - \underset{\sim}{j})\right)$$ ✓ ✓
$$= -3(\underset{\sim}{i} - \underset{\sim}{j})$$
$$= -3\underset{\sim}{i} + 3\underset{\sim}{j}$$ ✓

* Correct scalar multiple
* negates direction
* simplified answer (accept either)

Question 6 {1.2.10, 1.2.11, 1.2.12} **(12 marks)**

(a) Find the value of x given that the angle between vector $x\boldsymbol{i}+\boldsymbol{j}$ and $-\boldsymbol{i}-\boldsymbol{j}$ is 60^0.

(5 marks)

$\begin{pmatrix} x \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -1 \end{pmatrix} = |x\underset{\sim}{i} + \underset{\sim}{j}||-\underset{\sim}{i} - \underset{\sim}{j}| \cos 60°$ ✓ uses dot product

$-x-1 = \sqrt{x^2+1}\ \sqrt{2} \times \frac{1}{2}$ ✓ substitutes & evaluates

$-x-1 = \sqrt{x^2+1} \times \frac{\sqrt{2}}{2}$

$(-x-1)^2 = \frac{1}{2}(x^2+1)$ ✓ squares both sides

$x^2+2x+1 = \frac{1}{2}(x^2+1)$

$2x^2+4x+2 = x^2+1$

$x^2+4x+1 = 0$

✓ obtains quadratic

$x = \frac{-4 \pm \sqrt{16-4(1)(1)}}{2}$

$= \frac{-4 \pm \sqrt{12}}{2}$

$= \frac{-4 \pm 2\sqrt{3}}{2}$

$= -2 \pm \sqrt{3}$ ✓ both values

(b) Find the value of the constant n given that vector $2n\boldsymbol{i}+3\boldsymbol{j}$ is perpendicular to $-5\boldsymbol{i}+4j$.

(2 marks)

$-10n + 12 = 0$ ✓ uses dot product

$10n = 12$

$n = \frac{6}{5}$ ✓ or 1.2 correct value

(c) The vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are such that $|\boldsymbol{p}| = 14$, $|\boldsymbol{q}| = 16$ and $\boldsymbol{p}\cdot\boldsymbol{q} = -20$. Evaluate

i. $-4\boldsymbol{p}\cdot 2\boldsymbol{q}$ (2 marks)

$= -4 \times 2 \times p\cdot q$ ✓ substitutes $p\cdot q = -20$

$= -8 \times -20$

$= 160$ ✓ correct value

ii. $(\boldsymbol{q}+\boldsymbol{p})\cdot(\boldsymbol{p}-\boldsymbol{q})$ (3 marks)

$= \underset{\sim}{q}\cdot\underset{\sim}{p} - \underset{\sim}{q}\cdot\underset{\sim}{q} + \underset{\sim}{p}\cdot\underset{\sim}{p} - \underset{\sim}{p}\cdot\underset{\sim}{q}$

$= \underset{\sim}{p}\cdot\underset{\sim}{p} - \underset{\sim}{q}\cdot\underset{\sim}{q}$ ✓ simplified expression

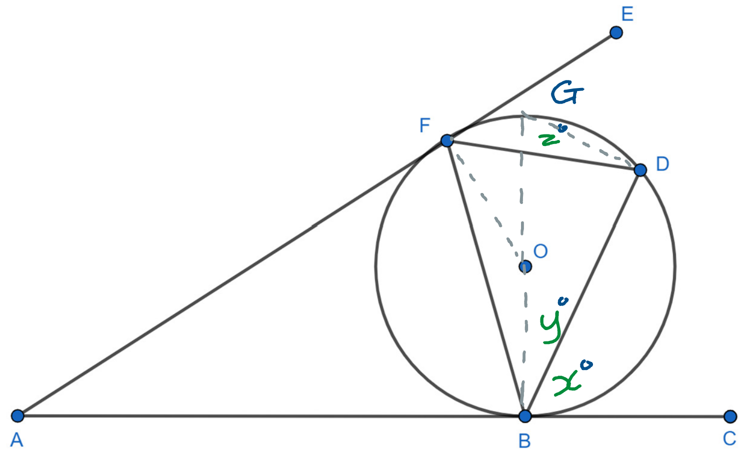
$= |\underset{\sim}{p}|^2 - |\underset{\sim}{q}|^2$

$= 14^2 - 16^2$ ✓ substitutes $= (14-16)(14+16)$

$= -2 \times 30 = -60$ ✓ correct value

Question 7 {1.3.6, 1.3.8, 1.3.10, 1.3.11} **(6 marks)**

Consider the diagram below which consists of a circle with the centre O, its two tangents AE and AC, and a chord BF which subtends the angle $\angle BDF$ on the circumference of the circle.



(a) Prove that $AF = AB$. (Note that it is not sufficient to simply quote a theorem that states they are equal.) (2 marks)

$OF = OB$ (radii) ✓ correct statements

OA (common) ✓ correct reasons

$\angle OFA = \angle OBA = 90°$ (radius $\perp$ tangent)

$\therefore \triangle FOA \cong \triangle BOA$ (RHS)

Thus $AF = AB$

(b) Prove that $\angle CBD = \angle BFD$. (Note that it is not sufficient to simply quote a theorem stating they are equal.) (4 marks)

draw a diameter from B through O to G

let $\angle DBC = x°$, $\angle GBD = y°$, $\angle BGD = z°$

$x° + y° = 90°$ (radius $\perp$ tangent) ✓ * correct statements

$z° + y° = 90°$ (angle in a semicircle) ✓ * correct theorems

$\therefore z° = x°$ ✓ correct deduction

Thus $\angle BFD = z°$ (angles in same segment)

$\therefore \angle CBD = \angle BFD$ ✓ correct theorem to support conclusion

Additional working space

Question number: ________

Additional working space

Question number: ________

Additional working space

Question number: ________